

Closing Wed: HW\_6A, 6B, 6C (7.3, 7.4, 7.5)

*Entry Task:*

Complete the square on each of these:

$$x^2 + 10x =$$

$$4x^2 + 4x + 3 =$$

$$14 - 8x - x^2 =$$

## 7.3 Trigonometric Substitution Summary

CASE	SUBSTITUTION
$\sqrt{a^2 - u^2}$	$u = a \sin(\theta)$ $du = a \cos(\theta) d\theta$ $\sqrt{a^2 - a^2 \sin^2(\theta)} = a \cos(\theta)$
$\sqrt{a^2 + u^2}$	$u = a \tan(\theta)$ $du = a \sec^2(\theta) d\theta$ $\sqrt{a^2 + a^2 \tan^2(\theta)} = a \sec(\theta)$
$\sqrt{u^2 - a^2}$	$u = a \sec(\theta)$ $du = a \sec(\theta) \tan(\theta) d\theta$ $\sqrt{a^2 \sec^2(\theta) - a^2} = a \tan(\theta)$

1. Trig Sub
2. Trig Integral (use 7.2 methods)
3. Triangle Trick

If you encounter a 'middle term'

$$\sqrt{ax^2 + \mathbf{bx} + c}.$$

Complete

Full Example:

$$\int \frac{x}{\sqrt{34 - 6x + x^2}} dx$$

1. Complete the square.
2. Do a trig sub problem.

## 7.4 Partial Fractions

*Motivation:* We will learn a process to break-up fractions like:

$$\frac{x^3 + 4x - 4}{x^2(x^2 + 4)} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^2 + 4}$$

If we can do this, then integration is “easy”:

$$\int \frac{x^3 + 4x - 4}{x^2(x^2 + 4)} dx$$

The only types of integrals in this section:

$$\int \frac{1}{2x + 5} dx = \frac{1}{2} \ln|2x + 5| + C$$

$$\int \frac{1}{(x - 4)^2} dx = -\frac{1}{x - 4} + C$$

$$\int \frac{1}{(x + 7)^3} dx = -\frac{1}{2} \frac{1}{(x + 7)^2} + C$$

$$\int \frac{1}{x^2 + 9} dx = \frac{1}{3} \tan^{-1} \left( \frac{x}{3} \right) + C$$

$$\int \frac{x}{x^2 + 9} dx = \frac{1}{2} \ln|x^2 + 9| + C$$

### Partial Fraction Decomposition

**Step 0:** Reduce, if needed.

**Step 1:** Factor Denominator and Write Decomposition Form

**Step 2:** Solve for Constants.

**Step 3:** Break up the integral, it will be a sum of the types above.

**Step 0:** Is the fraction *reduced*?

If yes, move to step 1.

If not, divide, then move to step 1.

*Quick review:* Do you remember how to reduce fractions with numbers? Show me:

$$\frac{576}{11} = ? + \frac{?}{11}$$

*For rational functions:*

*Reduced* means the highest power on top is strictly smaller than the highest power on bottom.

*Example:*

$$\int \frac{x^2 + x}{x + 3} dx$$

**Step 1:** Factor Denominator.

Write out decomposition below:

*Example:*

$$\int \frac{x+1}{x^2-4} dx$$

*i) Distinct Linear:*

$$\frac{x^2 - 3}{x(x-1)(x+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+4}$$

*ii) Repeated Linear:*

$$\frac{5+2x}{(x+3)(x-2)^3} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

*iii) Irreducible Quadratic:*

$$\frac{4x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

**Step 2:** Solve for A, B, C ....

*Example:*

$$\int \frac{x + 1}{x^3 + 3x^2} dx$$

*Example:*

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx$$

*Example:*

$$\int \frac{x}{x^2 + 4x + 5} dx$$



## How to integrate

A. Look for simplifications/substitutions

B. Products/Logs/Inverse Trig → BY PARTS

Sin/Cos/Tan/Sec combos → TRIG

Quadratic (under a radical) → TRIG SUB

Rational Function → PART. FRAC.

C. If nothing seems to work, substitution.

( $u = \text{inside}$ ,  $u = \sqrt{\quad}$ ,  $u = \text{trig}$ ,  $u = e^x$ )

*Examples of substitution:*

1.  $\int e^{\sqrt{x}} dx$

2.  $\int \frac{3}{x - 2\sqrt{x}} dx$

3.  $\int \frac{\cos(x)}{4 - \sin^2(x)} dx$

4.  $\int e^x \cos(e^x) \sin^3(e^x) dx$

How would you *start* these?

1.  $\int \tan^3(x) \sec(x) dx$

2.  $\int x^2 \ln(x) dx$

3.  $\int x \sqrt{5 - x^2} dx$

4.  $\int \frac{\sqrt{x^2 - 1}}{x^2} dx$

5.  $\int \frac{x^2 + 1}{x^2 - 2x - 3} dx$

6.  $\int x \tan^{-1}(x) dx$

7.  $\int \frac{dx}{\sqrt{4x^2 + 8x - 12}} dx$